Ion acoustic waves in plasmas with light and heavy ions

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The damping and dispersion of ion acoustic waves in a plasma with two ion species has been studied. We have investigated the transition from collisional to collisionless regimes using the Grad 13-moment approximation. Two branches of the ion dispersion relation have been found. The fast mode has a similar velocity as in plasmas with one ion species, but a much stronger damping rate due to the friction force between light and heavy ions. The effect of friction disappears when both ion species have the same charge-to-mass ratio. The heavy ions with a low degree of ionization strongly increase the magnitude of the fast ion acoustic wave damping. The slow ion acoustic wave is also found in the collisionless region, but this mode vanishes when its frequency approaches the ion-ion collision frequency. The stimulated Brillouin scattering from two ion species plasmas has been discussed in the context of recent experimental data. It is shown that stimulated Brillouin scattering reflectivity from the laser-produced plasma is dramatically increased when both ion species have the same charge-to-mass ratio.

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I. INTRODUCTION

Several different studies [1–4] have recently been conducted in order to describe ion acoustic waves (IAW) in the regime of plasma parameters where their frequencies are comparable to the ion-ion (i-i) collision frequency. These conditions are important in various aspects of plasma physics, including current laser-plasma interaction experiments. The well known asymptotic theories of ion response to IAW in collisional and collisionless limits are not adequate in this case of intermediate collisionality. We have shown [4], that the ion fluid theory of IAW must include frequency dependent transport coefficients in order to properly describe transition from a collisional to a weakly collisional regime. This theory has been derived for a fully ionized single-ion-species plasma using the generalized Grad moment expansion method. Typical laboratory plasma, however, usually involves more than one species of ion. Such a plasma can be a multiple-ion-species medium due to different levels of ionization which are generated during plasma production processes. The laser-produced plasmas, for example, involve multi-ion-component targets that have been used in experiments related to inertial confinement fusion research.

A standard, single-fluid approach to the description of ion transport in plasmas, which are composed of different ion species, introduces an average-ion model with mean mass $\bar{M} = (M_l n_l + M_h n_h)/n_i$ and charge $\bar{Z} = (Z_l n_l + Z_h n_h)/n_i$, where $n_i = n_l + n_h$ is the total ion density and subscripts l and h stand for light and heavy ions. The accuracy of this simple procedure has been recently investigated in the strongly collisional regime [5] showing significant underestimation of ion transport coefficients. Also, results obtained in the collisionless regime [6] have shown several new characteristics of IAW in multiple-ion plasmas, such as the existence of the slow ion acoustic mode. Our present study is motivated by these theoretical results and recent experiments on stimulated Brillouin scattering (SBS) [7]. The latter have shown values of reflectivities which depend on the ion composition of the laser-produced plasma.

In this paper we have developed the theory of IAW dispersion and damping in two-ion-component plasmas with a large temperature ratio, $\bar{Z}T_e/T_i \gg 1$, which is valid in a wide region of wavelengths and describes the transition from collisional to collisionless limits. This theory is based on the generalized Grad moment expansion method which has been successfully applied in our investigations related to single-ion-species plasmas [4]. It includes effects of frequency dependent ion electrical conductivity, thermal conductivity, and viscosity. In the strongly collisional regime, our model reproduces well hydrodynamic results of Ref. [5]. It extends, however, far beyond the collisional region and decribes a new IAW—the slow wave and dispersion characteristics of both modes in the region of weak collisions. We have applied our model to C_5H_{12} and C_5D_{12} plasmas, which have been investigated in recent SBS experiments [7], showing dramatic difference in IAW damping in the weakly collisional regime.

Our paper is organized in the following way. Section II contains a physical discussion of the friction between different types of ions. The two-ion-species fluid model is discussed in Sec. III. This discussion includes solutions of the linearized dispersion relation for fast and slow IAW. An approximate model of IAW dispersion and damping, which includes Landau damping, is constructed in Sec. IV. Finally, Sec. V contains a summary and comparison with experimental results.

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II. FRICTION BETWEEN DIFFERENT ION SPECIES

Friction between ions of different kinds is a distinct property of two-ion-species plasmas that does not occur in a one-ion-component system. In the latter case, only an inhomogeneous ion flow results in a dissipation of momentum and energy by viscosity. Uniform average velocities of two-ion-component plasmas can be dissipated by different-ion collisions via frictional force terms in the momentum transport equations. The effect of friction on light ions and IAW is very similar to the effect of high frequency resistivity due to electron-ion collisions on electrons in Langmuir waves.

To demonstrate the basic physics of this mechanism of IAW dissipation, we first consider a simplified model of a plasma with two ion species. The friction force between the light and heavy ions reads

$$\delta \mathbf{R} = g_0 \nu_{lh} n_l M_l (\delta \mathbf{u}_l - \delta \mathbf{u}_h) \ . \quad (1)$$

This force contributes to the linearized version of equations of motion for the average velocities of light, $\delta \mathbf{u}_l$, and heavy, $\delta \mathbf{u}_h$, ions:

$$M_l n_l \frac{\partial \delta \mathbf{u}_l}{\partial t} = -eZ_l \nabla \delta\phi - \delta \mathbf{R}, \quad (2)$$

$$M_h n_h \frac{\partial \delta \mathbf{u}_h}{\partial t} = -eZ_h \nabla \delta\phi + \delta \mathbf{R}. \quad (3)$$

In the expression (1) for the friction force, ν_{lh} stands for the collision frequency between different ions and a factor g_0 has been introduced to simplify comparison with results of more accurate theory discussed below. This factor accounts for the frequency dependence of the friction term, including low and high frequency limits of this force. The first term on the right hand sides of Eqs. (2) and (3) describes the force on ions due to the electric potential $\delta\phi$ which is responsible for the coupling between electrons and ions. For linear, low frequency ion waves the electric potential can be related to the electron density perturbation in the following way:

$$\delta n_e / n_e = e\delta\phi / T_e, \quad (4)$$

where T_e and n_e are the electron temperature and density. Expression (4) is derived from the electron equation of motion after neglecting electron inertia terms. The relation between density perturbations of different ion species follows from the quasineutrality condition

$$\delta n_e = Z_l \delta n_l + Z_h \delta n_h. \quad (5)$$

The continuity equations for both ion species

$$\frac{\partial \delta n_{l(h)}}{\partial t} + n_{l(h)} \mathrm{div} \delta \mathbf{u}_{l(h)} = 0 \quad (6)$$

complete our simple fluid model of a two-ion-species plasma. Equations (1)–(6) are linearized with respect to small perturbations of a plane wave form, i.e., $\propto \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$. They produce the following dispersion relation:

$$1 - \frac{k^2 c_f^2}{\omega^2} + ik^2 c_h^2 \frac{g_0 \nu_{lh}}{\alpha_h \omega^3} \left(\frac{\omega^2}{k^2 c_s^2} - 1 \right) = 0, \quad (7)$$

where $\alpha_{l(h)} = Z_{l(h)} n_{l(h)} / (Z_l n_l + Z_h n_h)$ and $c_{l(h)} = \sqrt{Z_{l(h)} T_e / M_{l(h)}}$ are the usual ion acoustic speeds for light, c_l, and heavy, c_h, ions. There are also two other natural combinations of characteristic velocities

$$c_f = \sqrt{\alpha_l c_l^2 + \alpha_h c_h^2} \qquad \text{and} \qquad c_s = \sqrt{\bar{Z} T_e / \bar{M}}, \quad (8)$$

which define the wave group velocity in the collisionless and collisional limits, respectively.

The real frequency Reω and damping rate $\gamma_i = -\,\mathrm{Im}\,\omega$ of the ion acoustic mode depend on the relation between Reω and the collision frequency ν_{lh}. In the strongly collisional regime, where $|\omega| \ll \nu_{lh}$, Eq. (7) yields

$$\mathrm{Re}\,\omega = kc_s, \qquad \gamma_i = \frac{\alpha_h}{2} \frac{c_s^2}{c_h^2} \frac{k^2 (c_f^2 - c_s^2)}{g_0 \nu_{lh}}. \quad (9)$$

This solution is very similar to the normal acoustic mode [8], with the exception of the damping term, which does not depend on the ion thermal velocity as the usual viscous damping does. This damping, Eq. (9), is approximately $\bar{Z}T_e/T_i$ times larger than the damping in one-ion-component plasmas. In the regime of weak collisionality, $|\omega| \gg \nu_{lh}$, Eq. (7) yields

$$\mathrm{Re}\,\omega = kc_f, \qquad \gamma_i = \frac{g_0 \nu_{lh}}{2\alpha_h} c_h^2 \left(\frac{1}{c_s^2} - \frac{1}{c_f^2} \right). \quad (10)$$

This mode has a larger group velocity, $c_f > c_s$, its damping is proportional to the collision frequency and does not depend on the wave number, similarly to the one-ion-component case [9,4]. It does not depend, however, on the ion thermal velocity, either and, therefore, is $\bar{Z}T_e/T_i$ times larger than that in one-ion-component plasmas. Expressions (9) and (10) derived here from a very simple model will be confirmed in the next section, where the more complete theory from the full set of transport equations for two-ion-component plasmas will be analyzed.

Based on the results (9) and (10) of the simple ion-fluid model, we can summarize the physics of two-ion-species response to IAW in the following way: (i) dispersion of the IAW exhibits significant modification when the transition from a collisional to a collisionless regime occurs. This is the result of differences in relative ion motion. In the collisional regime the ion acoustic frequency is small and both ion species move together with almost the same velocity. In the weakly collisional regime the heavy ions have not enough time to respond to IAW; therefore, density perturbations are associated predominantly with the motion of light ions. Hence, the velocity of the wave increases. (ii) The ion acoustic damping rate related to the friction force dominates the usual viscous term if the electron-to-ion temperature ratio is large, i.e., $Z_{l(h)} T_e / T_{l(h)} \gg 1$. However, the damping due to the friction force vanishes for ions with the same charge-to-mass ratio, $c_f = c_s$. In such plasmas an electric potential affects both ions in the same way and cannot accelerate them to different velocities. In this case Eqs. (9)

and (10) predict zero ion acoustic damping and another mechanisms of dissipation, such as ion viscosity, has to be taken into account.

III. TWO-ION-FLUID MODEL

In this section we derive more accurate expressions for the small amplitude ion response, which includes the effect of different-ion collisions together with collisional effects within each ion component. Our objective is to investigate IAW damping and dispersion in the intermediate collisional region and describe the transition between collisional and collisionless limits. We make use of the Grad moment expansion method which has already been applied in the case of one-ion-component plasmas [4]. This study [4] resulted in the theory describing ion response in the large range of collisionality, which agrees very well with Fokker–Planck simulations [2]. In the case of two-ion-species plasmas, the linearized Grad 13-moment equations read [10,11]

$$\begin{aligned} M_{l(h)}n_{l(h)}\frac{\partial \delta \mathbf{u}_{l(h)}}{\partial t} + \nabla(n_{l(h)}\delta T_{l(h)} + T_{l(h)}\delta n_{l(h)}) - \mathrm{div}\hat{\sigma}_{l(h)} &= -eZ_{l(h)}n_{l(h)}\nabla\delta\phi + \delta\mathbf{R}_{l(h)}, \\ n_{l(h)}\frac{\partial \delta T_{l(h)}}{\partial t} + \frac{2}{3}n_{l(h)}\delta T_{l(h)}\mathrm{div}\delta\mathbf{u}_{l(h)} + \frac{2}{3}\mathrm{div}\delta\mathbf{q}_{l(h)} &= 0, \\ \frac{\partial \delta\hat{\sigma}_{l(h)}}{\partial t} - n_{l(h)}T_{l(h)}\hat{W}(\delta\mathbf{u}_{l(h)}) &= \beta_{l(h)}\nu_{lh}\delta\hat{\sigma}_{l(h)}, \\ \frac{\partial \delta\mathbf{q}_{l(h)}}{\partial t} + \frac{5}{2}\frac{n_{l(h)}T_{l(h)}}{M_{l(h)}}\nabla T_{l(h)} - \frac{T_{l(h)}}{M_{l(h)}}\mathrm{div}\hat{\sigma}_{l(h)} &= \delta\mathbf{Q}_{l(h)}, \end{aligned} \tag{11}$$

where $\delta T_{l(h)}$, $\delta \mathbf{q}_{l(h)}$, and $\delta\hat{\sigma}_{l(h)}$ are the perturbations of temperature, heat flux, and viscous stress tensor of each ion component, respectively. The following definitions have been used:

$$\delta\mathbf{R}_l = -\delta\mathbf{R}_h = -M_l n_l \nu_{lh}\left(\delta\mathbf{u}_l - \delta\mathbf{u}_h - \frac{3}{5}\frac{\delta\mathbf{q}_l}{n_l T_l}\right), \qquad W_{ij}(\mathbf{u}) = \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\mathrm{div}\mathbf{u},$$

$$\delta\mathbf{Q}_l = \frac{3}{2}n_l T_l \nu_{lh}(\delta\mathbf{u}_l - \delta\mathbf{u}_h) - \rho_l\nu_{lh}\delta\mathbf{q}_l, \qquad \delta\mathbf{Q}_h = -\rho_h\nu_{hh}\delta\mathbf{q}_h,$$

$$\beta_l \equiv \beta = \frac{6}{5}\left(1 + \frac{Z_l^2 n_l}{Z_h^2 n_h}\right), \qquad \beta_h = \frac{6}{5}, \qquad \rho_l \equiv \rho = \frac{13}{10} + \frac{2\sqrt{2}}{5}\frac{Z_l^2 n_l}{Z_h^2 n_h}, \qquad \rho_h = \frac{4}{5}.$$

Note the light ion heat flux contribution to the friction force $\delta\mathbf{R}_l$ which was not taken into account in Eq. (1) of the preceding section. This new term defines factor g_0 in Eq. (1). The relative ion motion also provides a source of thermal energy $\delta\mathbf{Q}_l$ for the light ion component. Coefficients β and ρ simplify the notation and will be used in the latter part of this paper. The light-heavy (l-h) ion and heavy-heavy (h-h) ion collision frequencies are defined as follows:

$$\nu_{lh} = \frac{4\sqrt{2\pi}}{3}\frac{Z_l^2 Z_h^2 e^4 n_h \Lambda_{lh}}{\sqrt{M_l}T_l^{3/2}}, \qquad \nu_{hh} = \frac{4\sqrt{\pi}}{3}\frac{Z_h^4 e^4 n_h \Lambda_{hh}}{\sqrt{M_h}T_h^{3/2}},$$

where $T_{l(h)}$ are the ion temperatures and $\Lambda_{lh(hh)}$ are the Coulomb logarithms.

Together with the continuity equation (6), quasineutrality condition (5), and the equation for electron density perturbation (4), expressions (11) form a closed system of equations. Solving these equations in Fourier space, one can obtain the following dispersion relation:

$$[\omega^2 - \alpha_l k^2 c_l^2 - \Gamma_l k^2 v_{Tl}^2 + ig_l\nu_{lh}\omega][\omega^2 - \alpha_h k^2 c_h^2 - \Gamma_h k^2 v_{Th}^2 + ig_h\nu_{lh}\omega] = [\alpha_h k^2 c_l^2 + ig_l\nu_{lh}\omega][\alpha_l k^2 c_h^2 + ig_m\nu_{lh}\omega]. \tag{12}$$

Here, $v_{T\alpha} = \sqrt{T_\alpha/M_\alpha}$ $(\alpha = l, h)$ are the thermal velocities of light and heavy ions and the following notations have been used:

$$\Gamma_l = \frac{1}{3}\left(5 + \frac{4\omega}{\omega + i\beta_l\nu_{lh}}\right)\left[1 + \frac{2}{3}f_l\left(1 + \frac{9i\nu_{lh}\omega}{10k^2v_{Tl}^2}\right)\right], \quad \Gamma_h = \frac{1}{3}\left(5 + \frac{4\omega}{\omega + i\beta_h\nu_{hh}}\right)\left(1 + \frac{2}{3}f_h\right),$$

$$f_{l(h)} = \frac{k^2 v_{Tl(h)}^2}{\omega^2 + i\rho_{l(h)}\nu_{lh(hh)}\omega - k^2 v_{Tl(h)}^2}, \qquad g_l = 1 - \left(1 + \frac{9i\nu_{lh}\omega}{10k^2v_{Tl}^2}\right)f_l,$$

$$g_h = \frac{n_l M_l}{n_h M_h}\left(1 - \frac{9i\nu_{lh}\omega}{10k^2v_{Tl}^2}f_l\right), \quad g_m = \frac{n_l M_l}{n_h M_h}\left[1 - f_l\left(1 + \frac{4}{5}\frac{\omega}{\omega + i\beta_l\nu_{lh}} + \frac{9i\nu_{lh}\omega}{10k^2v_{Tl}^2}\right)\right],$$

where Γ_α corresponds to the ratio of partial ion specific heats and coefficients g_β ($\beta = l, h, m$) account for the modification of collision frequencies in different frequency domains.

In the limit of cold ions, $v_{Tl(h)} = 0$, dispersion equation (12) takes the form of Eq. (7) with the coefficient $g_0(\omega)$ defined as $g_0 = 1 - \frac{9}{10} i\nu_{lh}/(\omega + i\rho\nu_{lh})$. Thus, the two limiting expressions, Eqs. (9) and (10), correspond to the solution of Eq. (12), if $g_0 = 1 - (9/10\rho)$ is used in Eq. (9) for $\omega \ll \nu_{lh}$, and $g_0 = 1$ is substituted in Eq. (10) for $\omega \gg \nu_{lh}$. Also, the full dispersion relation for the ion acoustic wave in plasmas with cold ions can be obtained from Eq. (7) if the dependence $g_0(\omega)$ is included there. The full numerical solution is shown in Fig. 1, where the wave number is normalized by the effective ion collision length $l_s = c_s/\nu_{lh}$. The plasma composition C_5H_{12} is taken from experiment [7]. In order to demonstrate the effect of friction force more explicitly, two extremes are considered: fully and singly ionized carbon. In the fully ionized plasma the charge-to-mass ratio diffference between carbon and hydrogen is not too big. Because of that, the wave dispersion is weak and the damping rate is small. For singly ionized carbon one can see a much stronger dispersion of the phase velocity and a dramatic increase of the damping rate in the weakly collisional region, $kl_s \lesssim 1$.

A. Fast ion acoustic mode

The dispersion relation (12) describes two branches of ion acoustic waves: a fast wave with phase velocity larger than thermal velocities of both ion species and a slow wave with the phase velocity in between ion thermal velocities. Solving Eq. (12) for the plasma with $\bar{Z}T_e/T_i \gg 1$ and assuming that the wave damping is small, one can find the following expressions for frequency and damping rate of fast IAW:

$$\mathrm{Re}\,\omega^{(f)} = kc_s\left[1 + \frac{5}{6}\alpha_l \frac{v_{Tl}^2}{c_l^2} + \frac{\alpha_h}{6}\frac{v_{Th}^2}{c_h^2} \times \left(5 + \frac{4k^2c_s^2}{k^2c_s^2 + \beta_h^2\nu_{hh}^2}\right)\right],$$

$$\gamma_i^{(f)} = \frac{\alpha_h}{2}\frac{c_s^2}{c_h^2}\frac{k^2(c_f^2 - c_s^2)}{g_0\nu_{lh}} + \frac{2}{3}\alpha_l \frac{c_s^2}{c_l^2}\frac{k^2 v_{Tl}^2}{\beta_l \nu_{lh}} + \frac{2}{3}\alpha_h \frac{c_s^2}{c_h^2}\frac{k^2 v_{Th}^2 \beta_h \nu_{hh}}{k^2c_s^2 + \beta_h^2\nu_{hh}^2}, \quad (13)$$

where $g_0 = 1 - (9/10\rho)$. Equation (13) is valid in the hydrodynamic regime of strong collisions, where $|\omega| \ll \nu_{lh}$. It differs from the cold-ion solution of Eq. (9) by additional terms which are proportional to $v_{Tl(h)}$ and account for thermal effects. Note, that the first term in the expression for $\gamma_i^{(f)}$ coincides with the damping coefficient (9) found before from the simple ion-fluid model, whereas two additional terms are related to the viscosity damping of light and heavy ion components. There are no contributions to the damping from ion thermal conductivity which is negligible for high temperature ratios similar to single-ion-species plasmas [4]. In the regime of weak collisionality, $|\omega| \gg \nu_{lh}$, we have

$$\mathrm{Re}\,\omega^{(f)} = kc_f\left[1 + \frac{3}{2}\alpha_l \frac{c_l^2 v_{Tl}^2}{c_f^4} + \frac{\alpha_h}{6}\frac{c_h^2 v_{Th}^2}{c_f^4} \times \left(5 + \frac{4k^2c_s^2}{k^2c_s^2 + \beta_h^2\nu_{hh}^2}\right)\right],$$

$$\gamma_i^{(f)} = \frac{\nu_{lh}}{2\alpha_h}c_h^2\left(\frac{1}{c_s^2} - \frac{1}{c_f^2}\right) + \frac{2}{3}\alpha_l \frac{v_{Tl}^2 c_l^2}{c_f^4}\beta_l\nu_{lh} + \frac{2}{3}\alpha_h \frac{v_{Th}^2}{c_f^2}\frac{k^2 c_h^2 \beta_h \nu_{hh}}{k^2c_s^2 + \beta_h^2\nu_{hh}^2}. \quad (14)$$

Note that in Eq. (14) the ion acoustic damping depends very weakly on the wave number, and contributions from the light and heavy ion collisions enter into $\gamma_i^{(f)}$ additively.

Figures 2–4 demonstrate the dispersion and damping of the fast ion acoustic mode found from Eq. (12) for plasmas with different ion composition and temperature ratio. We assume equal ion temperatures $T_i = T_l = T_h \lesssim T_e$. Numerical solutions are in good agreement with the analytical formulas of Eqs. (13) when the temperature ratio is large, $T_e/T_i \gg 1$. The overall behavior of the IAW is basically the same for all plasmas, i.e., the phase velocity increases in the weakly collisional regime $0.1 < kl_s < 10$ and the magnitude of the phase velocity ω/k variations is larger if ions are getting hotter and heavy ions have lower charge. This is a distinct feature

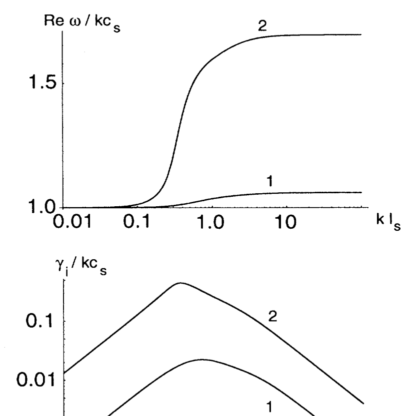

FIG. 1. The wave number dependence of the phase velocity $\mathrm{Re}\omega/kc_s$, and ion damping rate γ_i/kc_s, of the fast ion acoustic wave for C_5H_{12} plasma with fully ($Z_h = 6$, curves 1) and slightly ($Z_h = 1$, curves 2) ionized carbon in the limit of zero ion temperature. Wave numbers are normalized by the effective ion collision length $l_s = c_s/\nu_{lh}$.

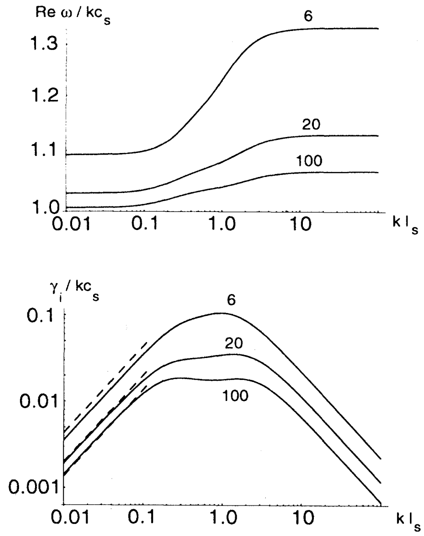

FIG. 2. Frequency and damping rates of the fast ion acoustic wave in the fully ionized C_5H_{12} plasma. Units are the same as in Fig. 1. Numbers near the curves show the temperature ratio T_e/T_i. Dashed lines represent the result of hydrodynamical theory of Ref. [5].

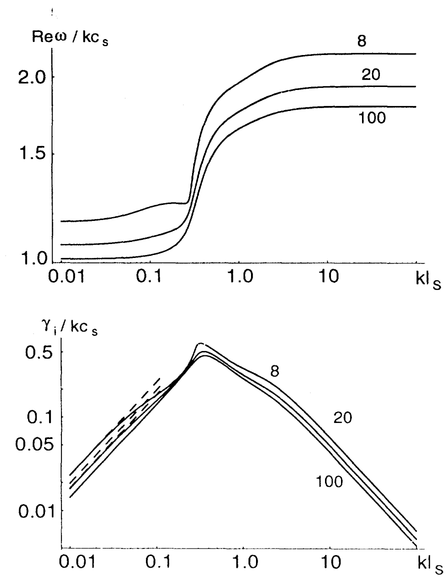

FIG. 4. Frequency and damping rates of the fast ion acoustic wave C_5H_{12} plasmas with single-ionized carbon. Notations are the same as in Fig. 2.

of two-ion-component plasmas; no significant modification in the ion acoustic phase velocity occurs in plasmas with single-ion species in this region of wavelengths. The relative wave damping achieves its maximum in this region, $kl_s \sim 1$. One can see (cf. Figs. 2 and 3) that in plasmas with equal charge-to-mass ratios of both ion species, the damping of the ion acoustic wave is small and it increases when the heavy ions have a smaller charge-to-mass ratio (cf. Fig. 3). The ion acoustic damping rate also increases when the temperature ratio decreases. In one particular example (Fig. 4), the wave damping becomes comparable to the wave frequency. It results in significant distortion of the dispersion curve due to the interaction of the ion acoustic branch of the dispersion equation (12) with other strongly damped nonoscillatory roots. In the strongly collisional region, $kl_s \lesssim 0.1$, our results compare well with the hydrodynamic theory of Ref. [5] (cf. dashed lines in Figs. 2–4). The small discrepancy in the case of plasmas with equal charge-to-mass ratio ions has to be attributed to the well known difference in numerical coefficients in the heat conductivity terms between the Grad 13-moment approximation and Braginskii's approach [11].

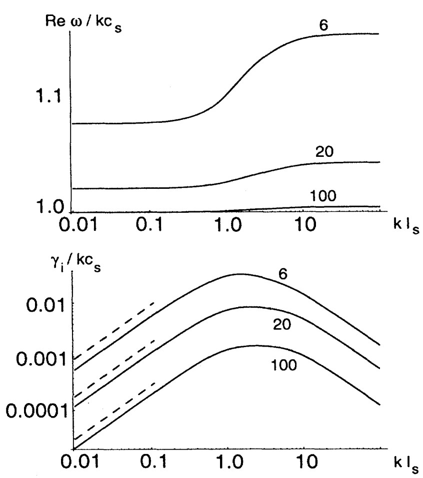

FIG. 3. Frequency and damping rates of the fast ion acoustic wave in the fully ionized C_5D_{12} plasma. Notations are the same as in Fig. 2.

B. Slow ion acoustic mode

In addition to the fast ion acoustic wave, Eq. (12) has several other roots, but almost all of them are strongly damped branches with $\mathrm{Re}\omega = 0$ and $\mathrm{Im}\omega \sim kc_s$. Because of that, they are not physically interesting. However, as the wave number increases in the weakly collisional region, $kl_s \gtrsim 1$, another oscillatory solution appears. Its frequency satisfies the conditions $kv_{Tl} > \omega > kv_{Th}$ and we call it the slow ion acoustic wave. By assuming that $\omega \gtrsim \nu_{lh}, \nu_{hh}$ one can obtain the analytical solution of Eq. (12) for this mode:

$$\mathrm{Re}\,\omega^{(t)} = kc_t, \qquad c_t^2 = \frac{\alpha_h v_{Tl}^2 c_h^2}{\alpha_l c_l^2 + v_{Tl}^2} + 3v_{Th}^2,$$
$$\gamma_i^{(t)} = \frac{\alpha_l c_l^2}{\alpha_l c_l^2 + v_{Tl}^2}\left[\frac{\nu_{lh} c_h^2}{2\alpha_h \alpha_l c_l^2} + \frac{\nu_{lh} c_h^2}{\alpha_l c_l^2 + v_{Tl}^2} \times \left(1 + \frac{2}{5}\frac{k^2 c_t^2}{k^2 c_t^2 + \beta_l^2 \nu_{lh}^2}\right) + \frac{2}{9}\frac{\beta_l \nu_{lh} k^2 c_t^2}{k^2 c_t^2 + \beta_l^2 \nu_{lh}^2} + \frac{2}{3}\frac{k^2 v_{Th}^2 \beta_h \nu_{hh}}{k^2 c_t^2 + \beta_h^2 \nu_{hh}^2}\right]. \tag{15}$$

The phase velocity of the slow mode c_t in Eq. (15) corresponds to the result of collisionless kinetic theory with the Debye screened light ions and cold heavy ions. In other words, in this wave the light ions behave in the same way as electrons in the fast ion acoustic wave. In the collisional region $\omega \lesssim \nu_{lh}$, the frequency of the slow mode becomes smaller than the damping rate and eventually it transforms into a strongly damped aperiodic mode with zero frequency. The interesting feature of the damping term in Eq. (15) is its weak dependence on the ion charge-to-mass ratio, in contrast to the fast ion acoustic wave (13). This is another demonstration of the fact that light ions respond to the electric potential in a different way than heavy ions. In the collisional region $\omega \lesssim \nu_{lh}$, the frequency of the slow mode becomes smaller than its damping rate and eventually it transforms into a strongly damped aperiodic mode with zero frequency.

The condition for relatively weak damping of the slow mode,

$$\frac{T_h M_l}{T_l M_h} \ll \mu \equiv \frac{Z_h^2 n_h M_l}{Z_l^2 n_l M_h} \ll 1, \tag{16}$$

assumes that the phase velocity of the mode is small compared to the thermal velocity of light ions and is large compared to the thermal velocity of heavy ions. For plasmas with comparable concentrations and temperatures of light and heavy ions, Eq. (16) cannot be satisfied if the heavy ions are almost fully ionized. Thus, for laser-produced plasmas with high electron temperature the slow wave has to be strongly damped. In fact, the weakly damped slow wave exists only in partially ionized plasmas with very heavy and/or cold ions. Formally, Eq. (12) does not forbid the excitation of the slow wave if $Z_h^2 n_h/M_h \lesssim Z_l^2 n_l/M_l$ and when $\omega \sim k v_{Tl}$. However, it is forbidden by strong Landau damping, which is not included in our quasihydrodynamic equations. Comparison of damping rates in Eqs. (13) and (15) shows that the collisional damping of the slow ion wave is always higher than that for the fast mode. Figure 5 demonstrates the damping and dispersion of the slow ion wave found from the numerical solution of Eq. (12) for $\mu = 0.3$. In the short wavelength region $kl_s \gg 1$, they are in good agreement with analytical formulas, Eqs. (15), which predict the weak dependence of the damping rate on the temperature ratio and the wave number. Figure 5 also shows that the slow ion wave originates from the aperi-

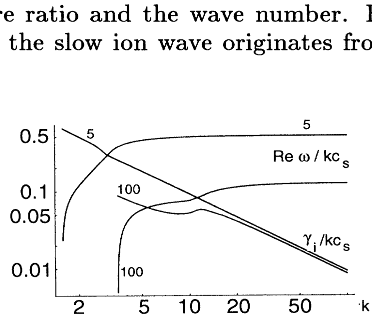

FIG. 5. Frequency $\mathrm{Re}\,\omega/kc_s$, and damping rate γ_i/kc_s, of the slow ion acoustic wave for C_5H_{12} plasmas with triple-ionized carbon for the temperature ratio $T_e/T_i = 5$ and 100.

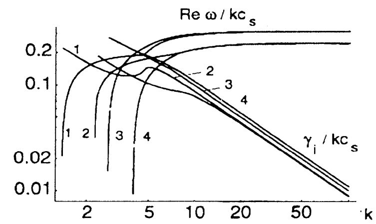

FIG. 6. Frequency and damping rates of the slow ion wave for $T_e/T_i = 15$ and different ion compositions: 1 – C_5H_{12} plasmas with triply ionized carbon ($\mu = 5/16$); 2 – C_5D_{12} plasmas with triply ionized carbon ($\mu = 5/8$); 3 – CH plasmas with triple ionized carbon ($\mu = 3/4$); 4 – C_5H_{12} plasmas with $Z_h = 5$ ($\mu = 125/144$). Units are the same as in Fig. 5.

odic branch in the region of intermediate collisionality, $kl_s \sim 1$. If the parameter μ, Eq. (16), increases, the damping of the slow wave also increases and the origin of the curve representing the slow ion wave moves further into the short wavelength region. This is shown in Fig. 6, where μ increases from curve 1 to curve 4 for different combinations of plasma parameters, $Z_h, n_h/n_l, M_l$.

IV. EFFECT OF ION LANDAU DAMPING

Our transport model accounts only for the collisional mechanism of ion wave dissipation. The correct description of collisionless Landau damping needs a more accurate kinetic treatment. In these cases, however, where the IAW damping is small compared to the wave frequency, one can incorporate the Landau damping term by adding it to the collisional damping that we have found in the preceding section. The conventional expressions for Landau damping do not account for the collisions which are destroying the wave-particle resonance when the mean free path of the resonance particle becomes comparable to the wavelength. In fact, an accurate description of the transition from the ion collisionless to ion collisional damping for the IAW has not yet been investigated. In Ref. [4] we have introduced the simple phenomenological function $\Psi(kl_{mfp}) = (kl_{mfp})^2/[q + (kl_{mfp})^2]$, which describes the destruction of the Ćerenkov resonance condition by ion-ion collisions. Here, l_{mfp} is the mean free path of a particle that resonantly interacts with the wave and therefore has a velocity v_r approximately equal to the wave phase velocity ω/k. Because the particle mean free path is proportional to the fourth power of the velocity, we have $l_{mfp} = l_{ii}(\omega/kv_{Ti})^4$, where $l_{ii} = v_{Ti}/\nu_{ii}$ is the mean free path of the thermal ion with respect to ion-ion collision. Comparison with kinetic simulations of the IAW damping [2] has shown that function $\Psi(kl_{mfp})$ with $q = 0.1$ properly accounts for the transition from collisionless to collisional IAW damping in one-ion-species plasmas.

We have implemented this phenomenological approach to the plasma with two ion species. This corresponds to the following form of the damping rate: $\gamma_i + \gamma_{Li} + \gamma_e$, where γ_i is the collisional damping found in the preceding section, γ_{Li} is the ion Landau damping contribution, and

γ_e accounts for the electron Landau damping. For the fast ion wave, the Landau damping contribution reads

$$\gamma_{Li}^{(f)} + \gamma_e = \sqrt{\frac{\pi}{8}}\Psi(kl_{mfp}^{lh})\frac{\omega^4}{k^3 v_{Tl}^3(1+\mu)}\exp\left(-\frac{\omega^2}{2k^2v_{Tl}^2}\right) + \sqrt{\frac{\pi}{8}}\Psi(kl_{mfp}^{hh})\frac{\mu\omega^4}{k^3 v_{Th}^3(1+\mu)}\exp\left(-\frac{\omega^2}{2k^2v_{Th}^2}\right) + \sqrt{\frac{\pi}{8}}\frac{\omega^4}{k^3v_{Te}^3(1+\mu)}\frac{M_l}{m_e\alpha_l Z_l}. \quad (17)$$

The first two terms in Eq. (17) account for the Landau damping related to light and heavy ions, and the frequency of the fast mode is given by Eq. (14). The third term is related to the electron Landau damping, v_{Te} is the thermal velocity of electrons with mass m_e. Functions Ψ describe the modification of the Landau damping due to ion-ion collisions, $l_{mfp}^{lh} = l_{lh}(\omega/kv_{Tl})^4$ and $l_{mfp}^{hh} = l_{hh}(\omega/kv_{Th})^4$ are the mean free paths of the resonant ions, $l_{lh} = v_{Tl}/\nu_{lh}$ and $l_{hh} = v_{Th}/\nu_{hh}$ are the mean free paths of the thermal ions. Similarly, for the slow mode in the case of $\mu < 1$ we have

$$\gamma_{Li}^{(t)} + \gamma_e = \sqrt{\frac{\pi}{8}}\Psi(kl_{lh})\frac{\omega^4}{k^3v_{Tl}^3\mu} + \sqrt{\frac{\pi}{8}}\Psi(kl_{mfp}^{hh})\frac{\omega^4}{k^3v_{Th}^3} \times \exp\left(-\frac{\omega^2}{2k^2v_{Th}^2}\right) + \sqrt{\frac{\pi}{8}}\frac{\omega^4}{k^3v_{Te}^3\mu}\frac{M_l}{m_e\alpha_l Z_l}. \quad (18)$$

The damping rates of the fast and slow ion acoustic waves, which include Landau damping contributions, are shown in Fig. 7. They are analyzed for $T_e/T_i \geq 5$. The damping of the slow wave is dominated by the light-ion Landau effect and it is much larger than the damping of the fast ion acoustic wave. In view of these results, the slow wave should be identified as a quasimode, because its frequency is comparable to the damping rate, as was noted before in Ref. [6]. The electron contribution to the wave damping also depends on the ion composition, but not so strongly as the terms with ion collisions. For example, the electron Landau damping of the fast ion acoustic wave in fully ionized C_5H_{12} plasmas exceeds the same value in C_5D_{12} plasmas by 1.2 times for $T_e/T_i = 100$ and 1.7 times for $T_e/T_i = 5$.

V. DISCUSSION

Our analysis has demonstrated the very sensitive dependence of the IAW damping rate on the plasma ion composition. This fact can help to clarify recent experimental observations [7] of the composition-dependent stimulated Brillouin scattering (SBS) from large size preformed plasmas. SBS has been observed from a plasma with electron density $n_e \approx 10^{21}$ cm^{-3}, electron temperature $T_e \approx 2$ keV, and characteristic scale length of about 0.1 cm. According to LASNEX simulations [7], the average ion temperature increases during the interaction time of 1.5 ns from $T_i = 0.2$ keV at the beginning of the pulse to 0.9 keV at the end. SBS in the backward direction from the interaction laser beam (the third harmonic of the Neodimium laser, $\lambda_0 = 0.35\,\mu$m) of intensity $I = (1-5)\times 10^{15}$ W/cm^2 has been observed during the first nanosecond. The most characteristic feature of these experiments is that the SBS reflectivity depends strongly on the ion composition. Namely, the SBS reflectivity from the C_5H_{12} target is approximately 5–10 times lower than that from C_5D_{12} plasma with the same parameters. Also, CO_2 plasma demonstrated approximately the same level of SBS reflectivity as C_5D_{12} plasma.

We can relate this observation to the composition-dependent damping of the ion acoustic wave that participates in backward SBS. Indeed, the major difference between C_5H_{12} plasma and C_5D_{12} or CO_2 plasma is the charge-to-mass ratio, i.e., Z/M is $\frac{1}{2}$ for C^{6+} and 1 for H^+, while Z/M is $\frac{1}{2}$ for D^+. Therefore, in C_5H_{12} plasma the ion acoustic wave exhibits an additional damping related to the friction force, as was discussed in Sec. II. Since the ion temperature is growing during the interaction time, we can anticipate that the ion acoustic damping will also change. In Fig. 8 we show the damping rate of the fast ion acoustic wave that participates in the Brillouin backscatter as a function of the electron-ion temperature ratio for C_5H_{12} and C_5D_{12} plasmas with the parameters of Ref. [7]. The damping of the ion acoustic wave in C_5D_{12} plasma is smaller than that in C_5H_{12} plasma in the whole temperature ratio range. For $T_e/T_i \sim 20$ the collisionality parameter is $kl_s \sim 10$ and the difference in the damping rate by a factor of four is mainly due to the effect of the friction force. As T_i increases, both damping coefficients grow. The damping of the ion acoustic

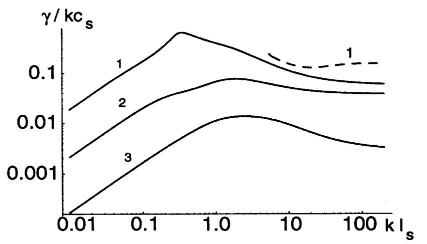

FIG. 7. Damping rate γ_i/kc_s, of the fast (solid lines) and slow (dashed curve) ion waves for plasmas with $T_e/T_i = 15$: 1 – C_5H_{12} with single-ionized carbon; 2 – C_5H_{12}, fully ionized carbon; 3 – fully ionized C_5D_{12} plasmas.

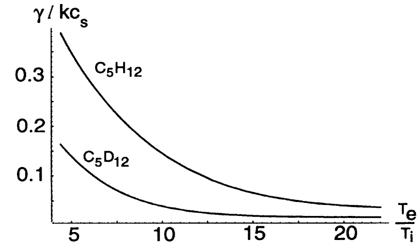

FIG. 8. Dependence of the fast ion acoustic wave damping on the temperature ratio for fully ionized C_5H_{12} and C_5D_{12} plasmas for the experimental conditions of Ref. [7]: $n_e = 10^{21}$ cm^{-3}, $T_e = 2.2$ keV, $k = 3.6\times 10^5$ cm^{-1}.

wave in C_5H_{12} plasma remains two times larger because the Landau damping related to H^+ ions is stronger than that related to the heavier deuterium ions.

Our discussion of SBS reflectivity observed in the hohlraum plasma experiments [7] is based on the assumption that the plasma scale length is so long (1–2 mm) that inhomogenities of density and flow velocity do not limit the SBS interaction length. The SBS gain in such experiments is defined by the collection of many hot spots created by the random phase plate (RPP) optics. In the first approximation, RPP hot spots can be treated as independent scattering centers [12]. Each of the hot spots has a length L_R approximately equal to the Rayleigh length of an ideal Gaussian beam of a given f number [12]. The stationary SBS reflectivity from an individual hot spot, $R \approx \epsilon \exp(2G)$, depends on the local gain coefficient $G = \gamma_0^2 L_R / c\gamma_s$ [13], where γ_0 is the SBS temporal growth rate, c is the speed of light, ϵ is the noise or seed level, and $L_R = 2\pi f^2 \lambda_0$ is the amplification length. For parameters of the experiment [7] the characteristic value of the amplification coefficient is about 10 for the average intensity 2×10^{15} W/cm^2, f=4, interaction length $L_R \approx 120$ μm, and $\gamma_s/kc_s = 0.1$. An increase of ion acoustic wave damping by a factor of two in this experiment will reduce the gain below a detection level. The importance of this estimate for SBS reflectivity is purely quantitative but it clearly indicates that the reflected signal from C_5H_{12} should have a shorter duration and smaller intensity than SBS from C_5D_{12} plasmas. Quantitative comparison with experiment [7], which should involve a more accurate description of the intensity distribution of the interaction laser beam and the SBS dynamics, is beyond the scope of this paper.

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